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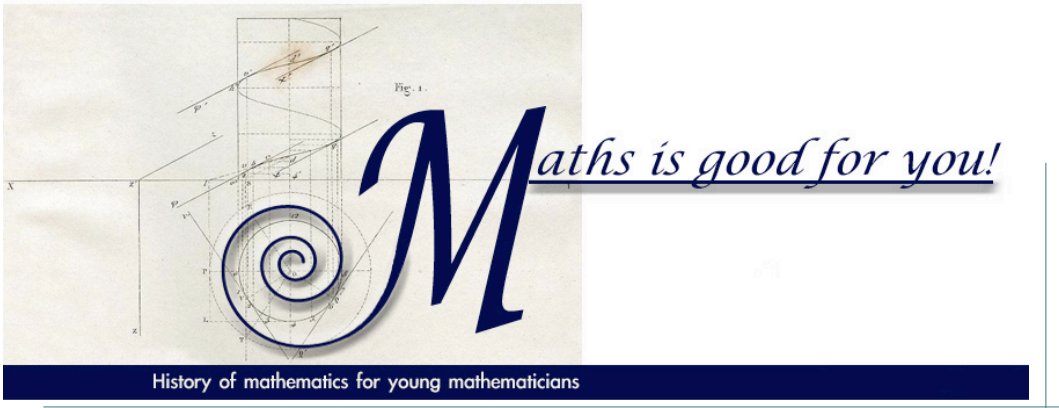
Worksheet on binary numbers

Teacher

Student

Class



Binary Numbers System

You can see more about the Binary Number System on
http://www.mathsisgoodforyou.com/topicsPages/number/basesystems.htm or

http://www.mathsisgoodforyou.com/topicsPages/number/binaryworking.htm

So let us write few numbers in the binary. First let us look back on our own decimal system and see how we determine the value of numbers. For example, you have a number 361. This number can be 'broken' down into

$3 \times 100 + 6 \times 10 + 1 \times 1$
in fact, the multipliers are powers of 10, so we can write this neater as
$3 \times 10^2 + 6 \times 10^1 + 1 \times 10^0 = 361$

In decimal system then, you have the powers of 10 – and you have 10 digits (0,1,2, etc.). In a binary system you only have powers of 2 (as you have two digits only: 0 and 1).

A number such as 10 written in binary would look like this

$1 \times 2^1 + 0 \times 2^0 = 2 + 1 = 2$ (in decimal notation). Let's get another one – number 101_2 (this small '2' means that this number is written in a binary system) would be

$101_2 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 4 + 0 + 1 = 5_{10}$ (this small '10' means that this number is now written in a decimal system).

Now you try

Number written in binary notation	...is a number written in decimal notation
10	
111	
1001	
1000 000	

So next time when you want something real bad, say that you will pay the person £1 000 000 or $1 000 000 but be careful not to say 'million' – just write the number down. It really is, in binary, worth only – how much?

You can try with a smaller table for each number to get the results. For example, look at the number 11101001. Now divide the digits and underneath them write the powers of 2 that relate to the digits of your number like this:

1	1	1	0	1	0	0	1
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

Now find all the values for the powers of 2 and multiply them by 1 or 0 (according to the upper row). You should get

2^7 128	2^6 64	2^5 32	2^4 16	2^3 8	2^2 4	2^1 2	2^0 1

$$128 \times 1 + 64 \times 1 + 32 \times 1 + 16 \times 0 + 8 \times 1 + 4 \times 0 + 2 \times 0 + 1 \times 1 = 233$$

In the similar way you can convert the numbers written in decimal to the binary system. Try to divide the number by 2 – you will either have a remainder (1) or not. So let us say we want to find how to write 31 in a binary system.

$31 = 15 \times 2 + 1$
$= (7 \times 2 + 1) \times 2 + 1$
$= ((3 \times 2 + 1) \times 2 + 1) \times 2 + 1$
$= (((1 \times 2 + 1) \times 2 + 1) \times 2 + 1) \times 2 + 1$

In this expression look what is multiplying 2s (as a number or remainder).

$(((1 \times 2 + 1) \times 2 + 1) \times 2 + 1) \times 2 + 1$

Now write that up as an expression which corresponds to multiplying with the powers of 2:

$(((1 \times 2 + 1) \times 2 + 1) \times 2 + 1) \times 2 + 1$

$1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 11111_2$ (1)

you can check that you have the right number from (1) $16 + 8 + 4 + 2 + 1 = 31$

Now try yourself:

Number written in decimal system	means number written in binary
23	
16	
106	

One can add, subtract, multiply and divide with binary numbers! But that is a task for another worksheet.