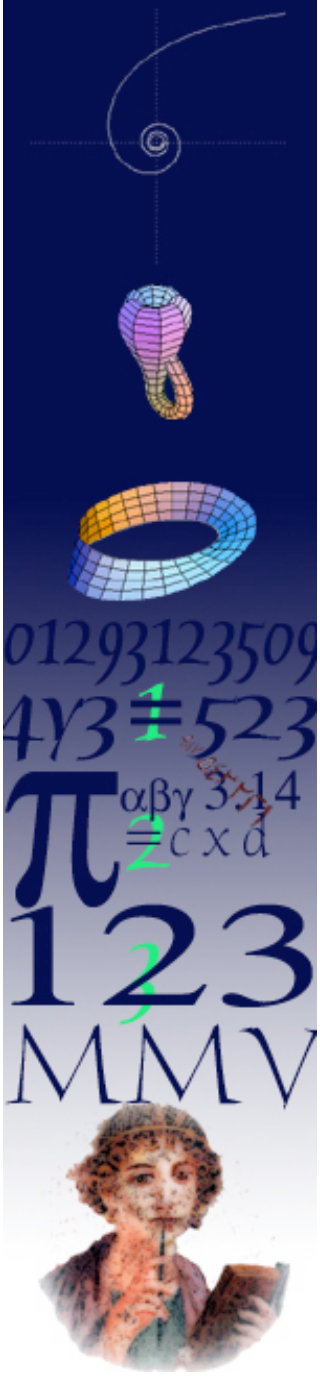
Maths is good for you!

History of mathematics for young mathematicians



Year 12 Pure Mathematics

C1 Coordinate Geometry 1

Edexcel Examination Board (UK)

Book used with this handout is Heinemann Modular Mathematics for Edexcel AS and A-Level, Core Mathematics 1 (2004 edition).

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Straight Lines

Every straight line can be represented algebraically in the form $y = mx + c$, where

m	represents the gradient of a line (its slope, steepness)
c	represents the y-intercept (a point where the line crosses the y axis)

Furthermore, there are several ways in which you can describe a straight line algebraically

Equation of a line

$y = mx + c$	Line gradient m, the intercept on y-axis is c
$y = mx$	Line gradient m, passes through the origin
$y = x + c$	Line gradient 1, makes an angle of 45^0 with the x-axis, and the intercept on the y axis is c
$y = k$	Line is parallel to the x axis, through $(0, k)$
$y = 0$	Line is the x axis
$x = k$	Line is parallel to the y axis, through $(k, 0)$
$x = 0$	Line is the y axis
$ax + by + c = 0$	General form of the equation of a straight line

Examples

	Equation of the line	Gradient	Coordinates of the y-intercept
1.	y = 8 – x		
2.	2y + 4x + 6 = 0		

3.		3	(0, 2)
4.		$-\frac{1}{2}$	$\left(0,-\frac{2}{3}\right)$

5. A line is parallel to the line $y=\frac{1}{2}x-3$ and its intercept on the y axis is (0, 1). Write down the equation of the line.

6. Find an equation of the line parallel to the line with equation $4x-2y=8$ and which passes through the point (2, -3)

7. The line $y=x-5$ meets the x axis at the point P. Work out the coordinates of P.

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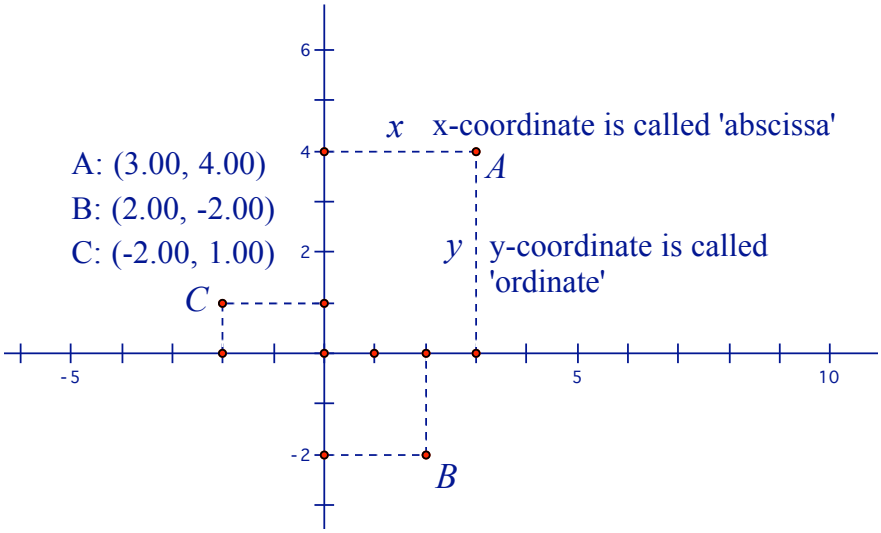
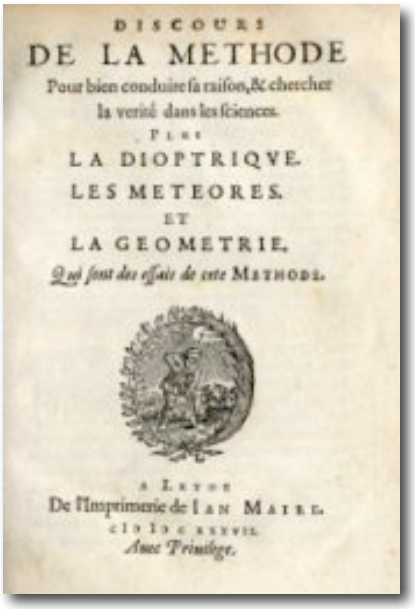
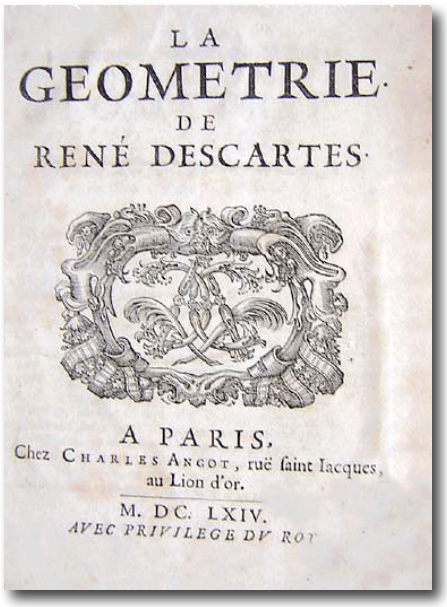
Historical background on coordinate geometry

Coordinate system, or *Cartesian coordinate system* as is sometimes known, is a rectangular system used to uniquely determine a point in two or three dimensional space by its distance from the origin of the coordinate system.

It gained its name from a French mathematician and philosopher René Descartes (1596-1650), most famously known for his work on merging algebra and geometry into algebraic geometry. He developed ideas about this system in his book – *Discourse on Method* (published in 1637), to which an appendix *The Geometry* (*La Géometrié*) was added trying to mathematically show the application of his philosophy. The book was also famous for the quote 'Je pense, donc je suis' (I think, therefore I am).




DISCOURS
DE LA METHODE
Pour bien conduire sa raison, & chercher la verité dans les sciences.
Plus
LA DIOPTRIQVE.
LES METEORES.
ET
LA GEOMETRIE.
Qui sont des essais de cette METHODE.
A LEYDE
De l'Imprimerie de IAN MAIRE.
Avec Privilege.

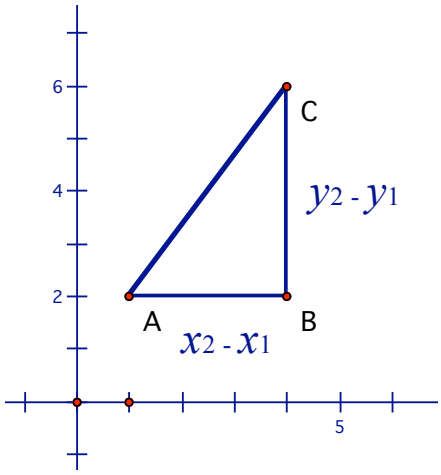

LA
GEOMETRIE
DE
RENÉ DESCARTES.
A PARIS,
Chez CHARLES ANGOT, ruë saint Iacques, au Lion d'or.
M. DC. LXIV.
AVEC PRIVILEGE DV ROY

Finding the gradient of a straight line joining two points

The gradient measures the steepness of the line.

It is defined as $\frac{increase'y'}{increase'x'}$, or $m = \frac{y_2 - y_1}{x_2 - x_1}$

When the gradient is 1, the line makes a 45^0 angle with either axes. If the gradient is 0, the line is parallel to the x axis.



Example

1.

Given the points on the line	Find its gradient
(2, 5) and (4, 8)	
(3, 7) and (7, -2)	
(2d, -5d) and (6d, 3d)	

2. The line joining (2, -5) to (4, a) has gradient -1. Find a.

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Equation of a straight line given the gradient and a point

If the point is given by its coordinates (x_1, y_1), and the gradient of a line is given as m, you can deduce the equation of that line.

You are using the formula for gradient, $m = \frac{y_2 - y_1}{x_2 - x_1}$, to derive a formula for the line itself. The coordinates of the point can be substituted, while the y_2 and x_2 need to remain (without the superscript numbers).

Then simply substitute the given values into

$$y - y_1 = m(x - x_1)$$

Examples

1. Find the equation of the line passing through (2, 11) with gradient 3.

2. Find the equation of a line with the gradient -2, which passes through a point (4, 1).

3. The line $y = 2x - 4$ meets the x axis at point A. Find the equation of the line with gradient 2/3 that passes through point A. Write your answer in the form $ax + by + c = 0$ where a, b, c are integers.

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The equation of a straight line given two points

When you have this kind of problem, you take that, as both points belong to the same line, the gradients at both points will be the same.

It makes sense therefore to say that $\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$

All you need to do in this case will be to substitute coordinates you have for the given points $A(x_1,y_1)$ and $B(x_2,y_2)$.

Examples

1. Find the equation of the line that passes through the points (3, 1) and (4, 7).

Find the equations of the lines passing through the points

2.	(3, -1) and (4, 4)	
3.	(-2, -4) and (-4, 8)	
4.	(7, -7) and (2, -2)	

5. The lines y = 4x – 7 and 2x + 3y – 21 = 0 intersect at point A. The point B has coordinates (-2, 8). Find the equation of the lines that passes through points A and B.

Write your answer in the form $ax+by+c=0$, where *a, b, c* are integers.

Parallel and perpendicular Lines

When two lines are parallel, their gradient is the same: $m_1 = m_2$

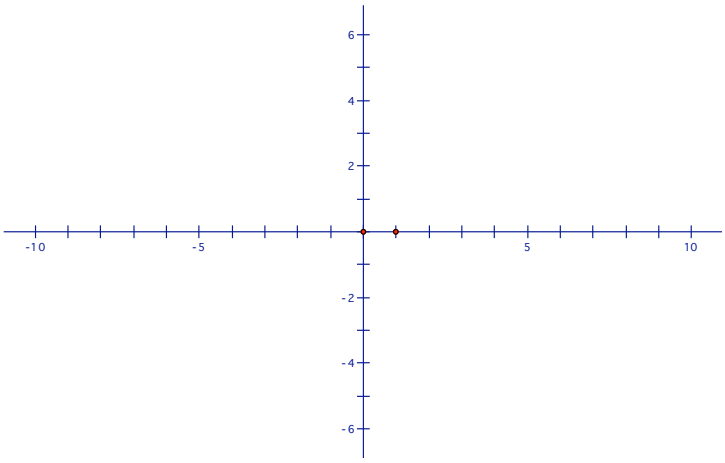
When two lines are perpendicular, their product equals -1: $m_1 m_2 = -1$.

Examples

1. Briefly sketch the lines and see whether the given lines are parallel or perpendicular

$y = 2x - 4$

$y = -\frac{1}{2}x + 3$



2. Find the gradient of the line perpendicular to
$y = 13x - 2$

3. Show that the lines are perpendicular by manipulating their gradients

$y = 4x$

$y = 7 - \frac{1}{4}x$

4. Show that the lines are parallel

$y = 1 - 6x$

$y = 7 - 6x$

5. Find an equation of the line that passes through the point A(1, 2) and is perpendicular to the line $y = 1 - 2x$. Write it in the form $ax + by + c = 0$.

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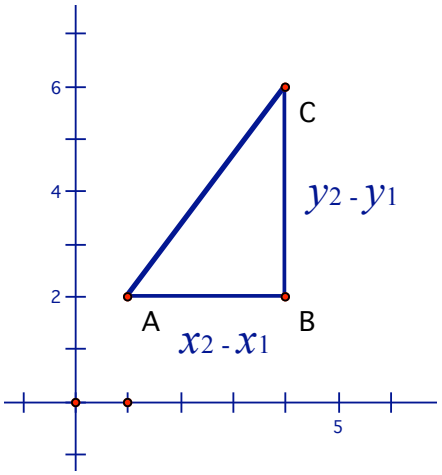
Length of line, its midpoint, and perpendicular bisector

The line length

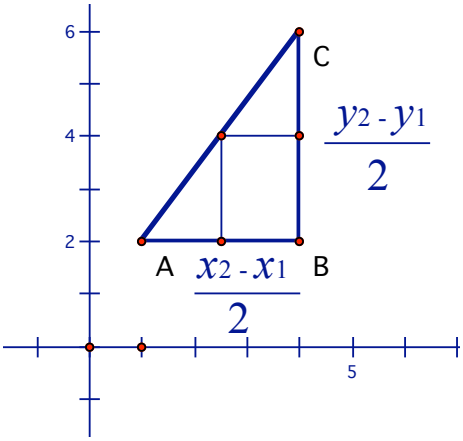
The length of the line segment joining two points will relate to their coordinates. Have a good look at the diagram



The length joining the point A and C can be found by using Pythagoras' Theorem:

$$AB^2 + BC^2 = AC^2$$

$$AC = \sqrt{AB^2 + BC^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Midpoint of a line

Mid-point of the line can be found by using the same principle

So the point between A and C will have the coordinates

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

If you know the midpoint, you can easily find the perpendicular bisector of a given line. This new line will go through the midpoint of the given line, and it will be perpendicular to it.

Example

Find the perpendicular bisector of the line joining the points (-3, 7) and (2, -10).

Answers

I'd love to do it for you – but simply don't have the time! Send me an email if you get stuck… ☺

Coordinate geometry test 2

This is fairly a simple part of the syllabus – just remember how to draw the main diagram and from there you can get all the formulae you need to use in coordinate geometry of a straight line.

Good way of revising is to complete all the mixed exercises which can be found at the end of the chapter on coordinate geometry. There is also a revision test to be downloaded at the same web-page where you got this booklet from.