



Maths is good for you!

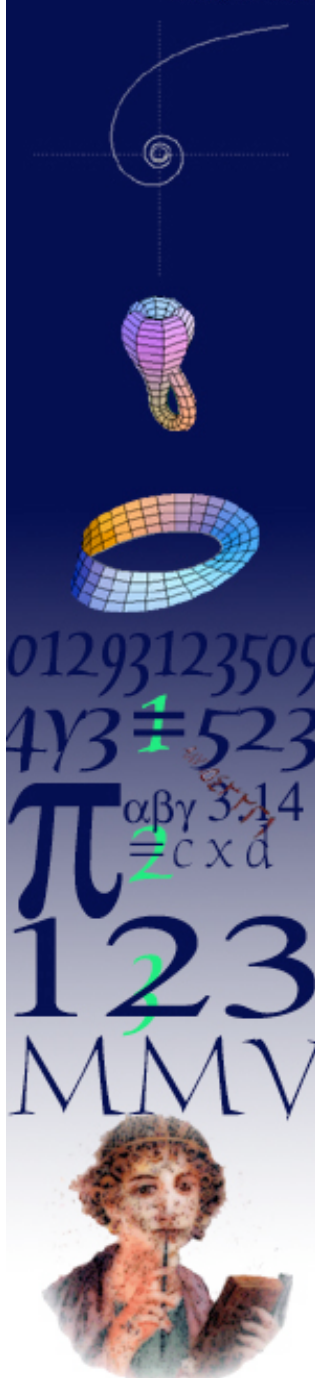
History of mathematics for young mathematicians

## Year 12 Pure Mathematics

### ALGEBRA 1

Edexcel Examination Board (UK)

*Book used with this handout is Heinemann Modular  
Mathematics for Edexcel AS and A-Level, Core  
Mathematics 1 (2004 edition).*



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## Indices

$$a^0 = 1$$

$$a^{-1} = \frac{1}{a}, \text{ and } \left(\frac{a}{b}\right)^{-1} = \left(\frac{b}{a}\right)$$

$$a^n \times a^m = a^{n+m}$$

$$a^n \div a^m = \frac{a^n}{a^m} = a^{n-m}$$

$$(a^n)^m = a^{n \times m}$$

$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

$$a^m \times b^m = (ab)^m, \text{ and } \left(\frac{a}{b}\right)^m$$

## Examples

1. $25^{\frac{1}{2}}$	2. $(-8)^{\frac{1}{3}}$
3. $\left(\frac{1}{4}\right)^{\frac{3}{2}}$	4. $(-1)^{\frac{1}{5}}$
5. $7^0$	6. $\frac{2^0 \times 3^{-2}}{5^{-1}}$
7. $8^{-\frac{1}{3}}$	8. $\left(\frac{8}{27}\right)^{-\frac{1}{3}}$

## Using and manipulating *surds*

### Introduction – number sets

Real numbers ( $\mathbb{R}$ ) are all numbers that correspond to some number on a number line (whether whole – integers, or decimal numbers, positive or negative).

The set of real numbers can be separated into two subsets: rational ( $\mathbb{Q}$ ) and irrational (no sign for them) numbers. All rational numbers can be expressed as fractions of two integers, whereas the irrational numbers cannot.

Natural numbers are positive counting numbers (1, 2, 3, etc.). They are denoted  $\mathbb{N}$ . Integers are whole numbers, whether positive or negative, denoted  $\mathbb{Z}$  (and there are two subsets to this – positive integers,  $\mathbb{Z}^+$ , and negative,  $\mathbb{Z}^-$ ).

Roots that cannot be expressed as rational numbers are called SURDS. Surds are expressions containing an irrational number – they cannot be expressed as fractions of two integers. Examples are  $\sqrt{2}, \sqrt{11}, \sqrt{10}$ . When asked to give an exact answer, leave the surds in your answer (although you may have to manipulate and/or rationalise the denominator) as you can never give an exact value of an irrational number.

### Rationalising the denominator

For historical reasons, you need to somehow always ‘get rid’ of the surd if it is in the denominator (see the historical note on surds). There are several techniques of manipulation you can use.

When multiplying or dividing with surds, this may be helpful

$$\begin{aligned}\sqrt{a} \times \sqrt{b} &= \sqrt{a \times b} \\ \frac{\sqrt{a}}{\sqrt{b}} &= \sqrt{\frac{a}{b}} \\ \sqrt{a} \times \sqrt{a} &= \sqrt{a^2} = a\end{aligned}$$

When you get the expression which looks something like this:

$$\frac{1}{\sqrt{5}-2}$$

you need to think how to get the surd in the numerator and have denominator without any surds. So you may want to apply the difference of squares:

$$(x-y)(x+y) = x^2 - y^2$$

This can also be applied to the case when one, or both, of the members of the brackets are square roots (or surds):

$$(\sqrt{x} - \sqrt{y})(\sqrt{x} + \sqrt{y}) = x - y$$

### Common mistake

**Never, ever** split the sum or difference of an expression like  $\sqrt{x+y}$  into  $\sqrt{x} + \sqrt{y}$ . You can't do that because you have two different numbers in an expression which is of the form  $(x+y)^{\frac{1}{2}}$ . And as with any other powers of expressions within the brackets you can't just apply the powers to the members of the brackets, but you must imagine these bracketed expressions multiplying each other as many times as the power says.

What you CAN do is break a number or a product under a root into its factors, some of which will hopefully be numbers that you will be able to take outside of the root. For example,

$$\sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$$

or

$$\sqrt{8x^3} = \sqrt{4 \times 2 \times x^2 \times x} = 2x\sqrt{2x}$$

### Examples

Express these in terms of the simplest possible surds

1.  $\sqrt{10}$

2.  $\frac{\sqrt{63}}{3}$

3.  $\sqrt{72}$

4.  $\sqrt{98}$

**3 rules to remember...**

If a fraction is of the form	Multiply top and bottom by	To get...
$\frac{c}{\sqrt{a}}$	$\sqrt{a}$	$\frac{c\sqrt{a}}{a}$
$\frac{c}{a + \sqrt{b}}$	$a - \sqrt{b}$	$\frac{c(a - \sqrt{b})}{a^2 - b}$
$\frac{c}{a - \sqrt{b}}$	$a + \sqrt{b}$	$\frac{c(a + \sqrt{b})}{a^2 - b}$

**Examples**

1.  $\frac{1}{2\sqrt{5}}$

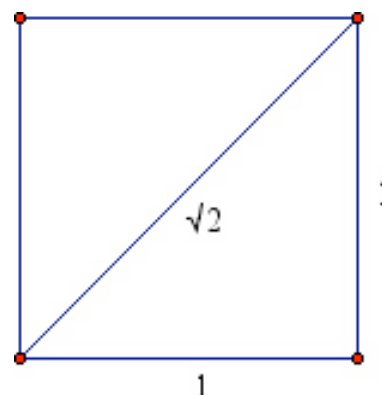
2.  $\frac{1}{\sqrt{5} + \sqrt{3}}$

3.  $\frac{1 + \sqrt{2}}{3 - \sqrt{5}}$

## Historical notes on surds

The meaning of the word ‘surd’ is twofold – in mathematics it refers to a number that is partly rational, partly irrational; in phonetics it refers to sounds uttered without vibration of the vocal cords – and so voiceless, ‘breathed’. So if you put the two together, you get that surds are ‘mute’ or ‘voiceless’ numbers. This is, actually, not a totally loopy statement: as you may know, certain ratios of numbers give pleasing sounds and have been described as far back as the Pythagoras’ time. Pythagoreans were credited with the discovery that the sound produced by an instrument is defined by the ratio between the length of a string which produced it and the (in some way, for example by pressing a finger against the string) shortened length of that string. So for example one speaks of a major third (4:5), or a minor third (5:6) in music, and this has a mathematical description – 4:5 or 5:6. What is important to understand now is the fact that the irrational numbers are also incommensurable with other numbers. Look at the square below:

The side of the square is 1, and its diagonal will be  $\sqrt{2}$ . As  $\sqrt{2}$  is an irrational number, its decimal will never end (and it does not have a repeating pattern). This means that you cannot express the ratio between 1 and  $\sqrt{2}$  as a rational number and so you will not get a ‘clear’ ratio – in fact if you attempt to get a sound which is produced from the ratio such as this you will not get a pleasant, clear, sound, merely a shshssssshh of a rather unpleasant quality.



In fact, the  $\sqrt{2}$  has a rather bloody history to it; it is claimed that the Pythagorean who discovered the property of  $\sqrt{2}$ , namely that it cannot be expressed as ratio of two rational numbers, was drowned at sea. This is because Pythagoreans believed that all of the universe can be described through numbers and their ratios – discovery that some numbers cannot be described as ratios of two other numbers caused them a great deal of confusion.

It is known that Al-Khwarizmi identified surds as something special in mathematics, and this mute quality of theirs. In or around 825AD he referred to the rational numbers as ‘audible’ and irrational as ‘inaudible’. It appears that the first European mathematician to adopt the terminology of surds (*surdus* means ‘deaf’ or ‘mute’ in Latin) was Gherardo of Cremona (c. 1150). It also seems that Fibonacci adopted the same term in 1202 to refer to a number that has no root. In English language the ‘surd’ appeared in the work of Robert Recorde’s *The Pathwaie to Knowledge*, published in 1551. Robert Recorde was a famous writer of popular mathematics and science books, aimed at those people who wanted to learn more of the subjects and could not attend schools: so workmen, merchants and the like bought Recorde’s books to learn practical knowledge related to mathematics or science.

Because it is impossible to evaluate surd to an absolutely accurate value, people were in the past (before the calculators and computers) using books which gave calculations of surds to a great number of decimal places. Surds in the denominator could not, however, be calculated in this way, as you cannot possibly exhaust all possibilities on how surds will appear. That is why ‘rationalising’ the denominator became such a necessary technique.

## Simplifying Expressions

This is revision of GCSE work. Work through the examples on Pages 1, 3 and 4 and complete Exercise 1C and 1D.

## Factorising Quadratics

Expand the expression  $(3x^2 - 1)(x + 4)$

These two expressions are identically equal, ie they are true for ALL values of x. The identity symbol is  $\equiv$

Expand these

$$(x + y)^2 \equiv$$

$$(x - y)^2 \equiv$$

The *factors* of the expressions are on the left hand side.

The process of taking an expression and writing it as a product of its factors is called ***factorisation***. E.g. you factorise

$$\begin{aligned} & a^2 - b^2 \\ & \text{to get} \\ & a^2 - b^2 \equiv (a + b)(a - b) \end{aligned}$$

## Useful Products and Factorisations for Algebraic Manipulation

These are to be memorised if you can – they can be very useful to you.

$$\begin{aligned} (a + b)(a - b) &= a^2 - b^2 \\ (x + a)(x + b) &= x^2 + (a + b)x + ab \\ (a + b)^2 &= a^2 + 2ab + b^2 \\ (a - b)^2 &= a^2 - 2ab + b^2 \\ (a + b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3 \\ (a - b)^3 &= a^3 - 3a^2b + 3ab^2 - b^3 \\ a^3 + b^3 &= (a + b)(a^2 - ab + b^2) \\ a^3 - b^3 &= (a - b)(a^2 + ab + b^2) \end{aligned}$$

### Common mistake

Don't ever do something like  $(a+b)^2 = a^2 + b^2$  because you have missed the point that  $(a+b)^2$  actually means  $(a+b)(a+b)$ , which will give you, as above,  $(a+b)^2 = a^2 + 2ab + b^2$ .

### Examples

Factorise the following expressions. First look for common factors (a factor that occurs in each term). If you get a quadratic with three terms, then look at the coefficients first:

$$ax^2 + bx + c$$

In the above expression the coefficients are  $a$ ,  $b$ , and  $c$ .

Follow this procedure:

1. look at the product of coefficients  $a$  and  $c$  and identify it
2. identify the factors of this product which, when added, give coefficient  $b$
3. you have it – let us say these two numbers are  $\alpha_1$  and  $\alpha_2$ . Then your factorised expression should look like

$$(x - \alpha_1)(x - \alpha_2)$$

Now you try

1	$8x^2 + 5x$
2	$25x^3 + 10x^2 - 15x$
3	$9x^2 - 4$

## Solving Quadratic Equations

Solving equation means finding a value for the unknown quantity which satisfies that equation. The same applies to quadratics as to linear equations – only that for quadratics you may have three possibilities:

- two solutions to the equation
- one solution
- no solutions.

You can use the factorisation technique you just practiced to identify roots of an equation. In other words if  $(x - \alpha_1)(x - \alpha_2) = 0$ , that means that either

$$(x - \alpha_1) = 0$$

in which case  $x = \alpha_1$

or

$$(x - \alpha_2) = 0$$

in which case

$$x = \alpha_2$$

and so the roots of that equation are  $\alpha_1$  and  $\alpha_2$ .

You try solving the following equations:

1. $x^2 - 4 = 0$
2. $x^2 - 9x - 36 = 0$
3. $2x^2 - 5x - 18 = 0$
4. $(2x - 1)^2 = 25$

## Completing the Square

There is another way of solving quadratic equations – it is called ‘completing the square’.

Perfect Squares are expressions such as these:

$$(a + b)^2 \equiv a^2 + 2ab + b^2$$

$$(a - b)^2 \equiv a^2 - 2ab + b^2$$

Every quadratic expression can be written as

$$ax^2 + bx + c$$

Let's say you have a usual quadratic of the form

$$ax^2 + bx + c = 0, \text{ where } a = 1$$

You need to use half of  $b$ , so let us introduce a value  $p$  such that  $p = \frac{b}{2}$ .

Re-write your equation as

$$(x + p)^2 - p^2 + c = 0$$

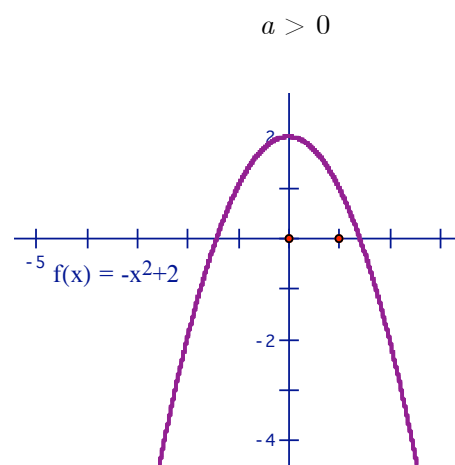
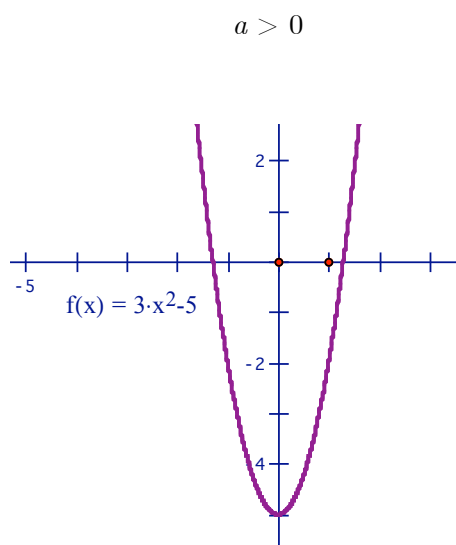
Because neither  $p^2$  nor  $c$  have  $x$  ‘attached’ to them, you can tidy the equation up and that will be the completed square of the original equation.

You can also use the completed square form to sketch the graph of a quadratic faster. The information given by the completed square form is

$$y = a(x + p)^2 + q$$

This is useful not only for finding the zeros (roots) of the quadratic, but also because you can then easily sketch it.

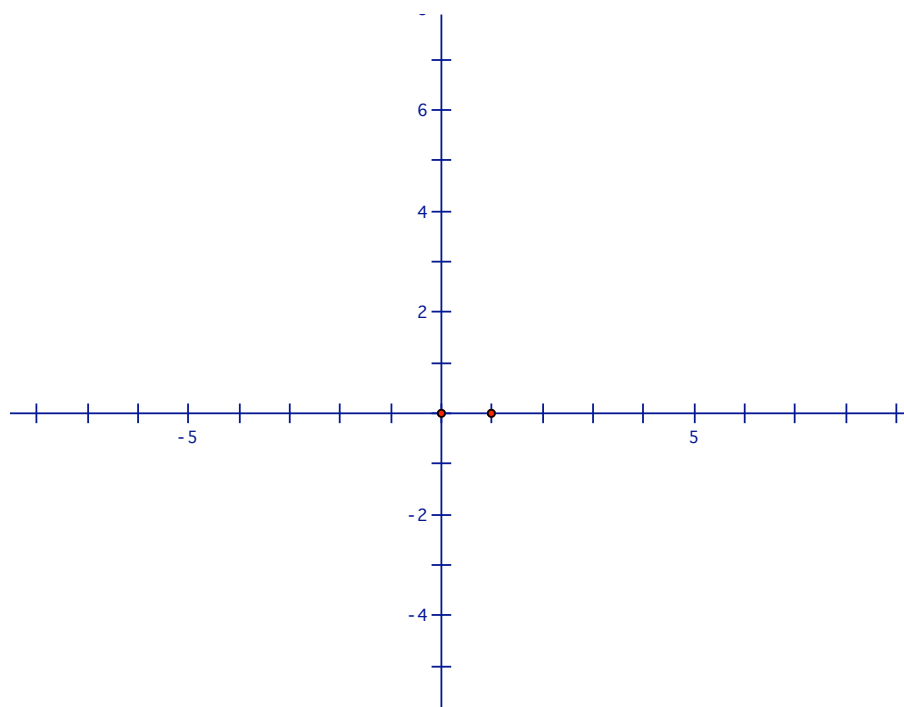
The vertex of parabola will have coordinates  $(-p, q)$ ; its axis of symmetry will be  $x = -p$ . It will be  $\cup$  shaped if  $a > 0$ , and  $\cap$  shaped if  $a < 0$ .



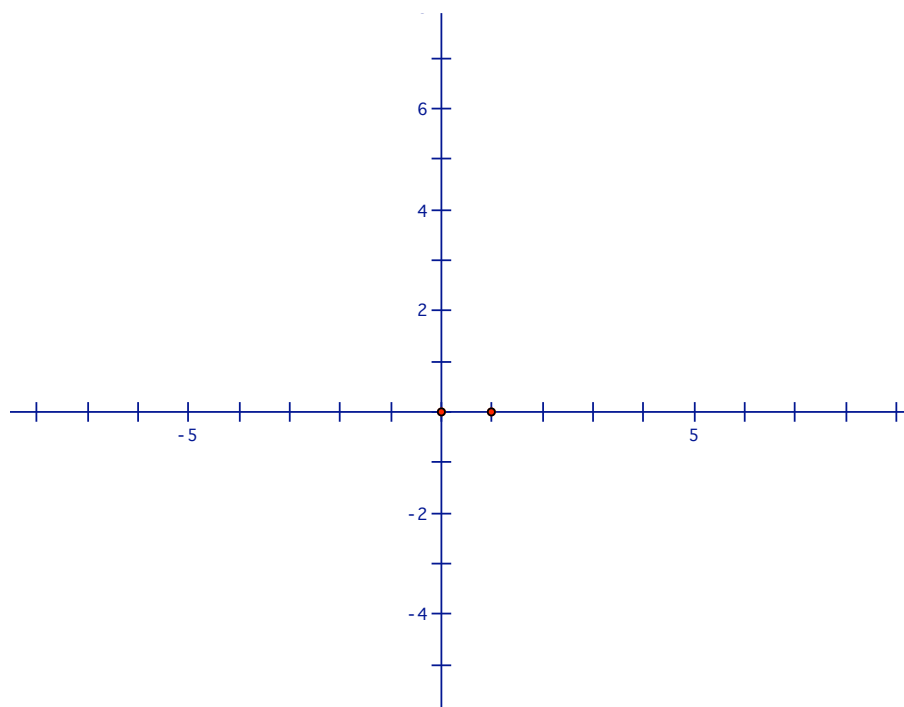
**Examples**

Complete the square and briefly sketch the quadratics:

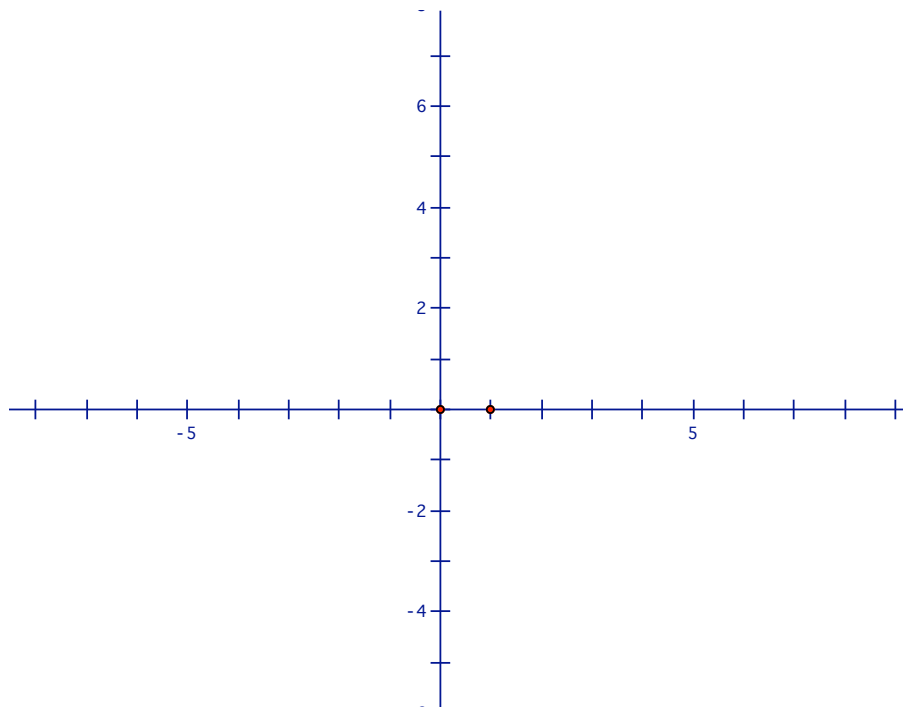
1.  $x^2 + 8x =$



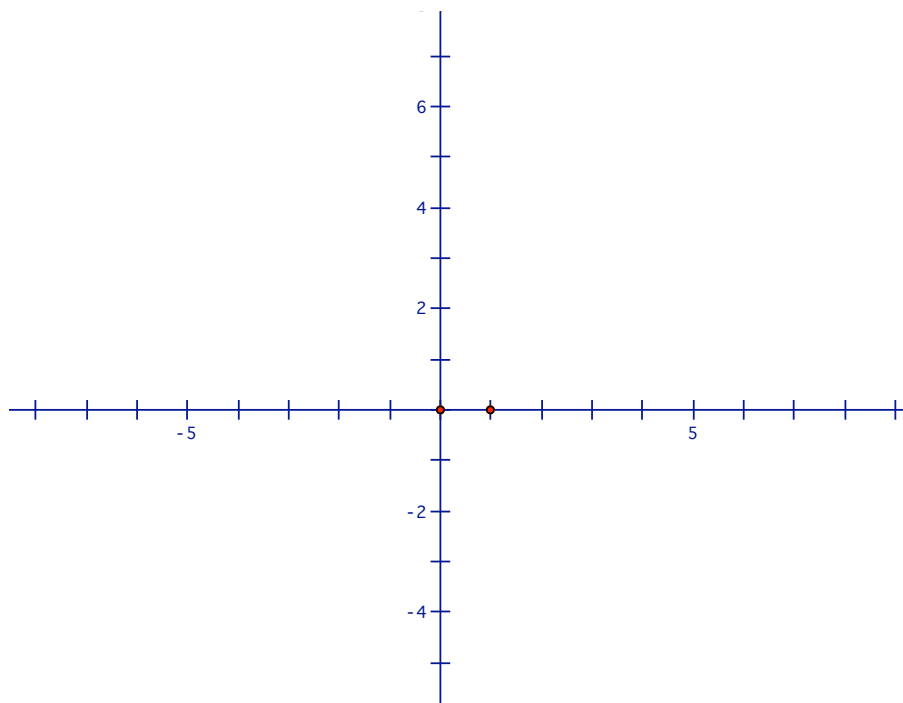
2.  $x^2 + 12x =$



3.  $2x^2 - 10x =$



4.  $x^2 + 3x =$



Page 18 Ex 2C and page 19 Ex 2D

## *History of the technique known as ‘completing the square’*

General quadratic equations of the form  $ax^2 + bx + c = 0$  were known to Egyptian, Babylonian, Chinese, Indian and Greek mathematicians. Arab mathematicians made progress on the solutions to these equations, in particular the famous al-Khwarizmi (Muhammad ibn Musa al-Khwarizmi who lived approximately between 780 and 850 AD in Baghdad). Al-Khwarizmi worked at the House of Wisdom in Baghdad and wrote a famous book *Hisab al-jabr w'al-muqabala*, from which we derive the meaning and the name of **algebra**.

One of the famous problems which introduces the ‘completion of the square’ was described by al-Khwarizmi in this book as follows:

*What must be the square which, when increased by ten of its own roots, amounts to 39?*

The equation can actually be described in modern-day symbols like this

$$x^2 + 10x = 39$$

Al-Khwarizmi solved this by adding 25, or  $5^2$  to both sides of the equation (he got 5 as  $10/2$ ) so that the left-hand side of the equation becomes a perfect square. In this case actually so does the right-hand side, but that is not always the case, nor is it necessary.

So we get

$$\begin{aligned} x^2 + 10x + 25 &= 39 + 25 \\ (x + 5)^2 &= 64 \end{aligned}$$

This can now be easily solved by taking the square root of both sides

$$x + 5 = \pm 8$$

or

$$x_{1,2} = \pm 8 - 5$$

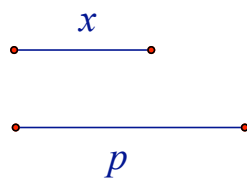
so

$$x_1 = 3$$

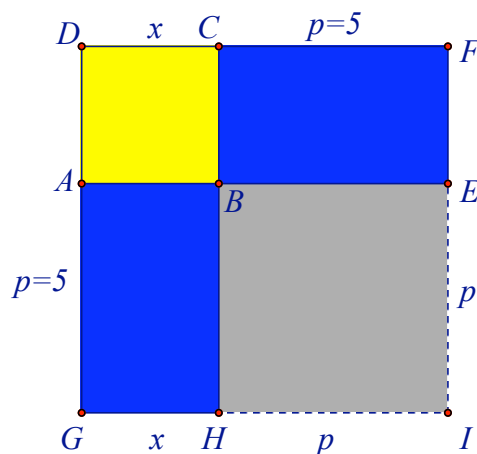
$$x_2 = -13$$

Al-Khwarizmi did not take into account the negative solution (that come some centuries later) so the only solution to this equation is for  $x = 3$ .

This is the diagram which represents the problem:



$$p = 5 \text{ cm}$$



$$\text{Area ABCD} = 9 \text{ cm}^2$$

$$\text{Area BEFC} = 15 \text{ cm}^2$$

$$\text{Area AGHB} = 15 \text{ cm}^2$$

$$\text{Area HIEB} = 25 \text{ cm}^2$$

$$(\text{Area ABCD}) + (\text{Area BEFC}) + (\text{Area AGHB}) + (\text{Area HIEB}) = 64 \text{ cm}^2$$

$$x = 3 \text{ cm}$$

$$(\text{Area ABCD}) + 2 \cdot (\text{Area BEFC}) = 39 \text{ cm}^2$$

## Solving Quadratic Equations using the Quadratic Formula

So far you have found out how to solve quadratics using factorisation and by completing the square. By far the easiest is probably the third way of solving quadratics – the one we will do now – as all that is required from you to do is to replace coefficients from the quadratic in a formula in order to get solutions. The formula in question is called *quadratic* formula; it is derived from the completed square form of a quadratic.

### Deriving the quadratic formula - proof

$$ax^2 + bx + c = 0$$

First divide both sides with  $a$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Complete the square

$$\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = 0$$

which means that

$$\left(x + \frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

rearrange the right-hand side of the equation, to include it all in one algebraic fraction

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

take the square root of both sides, you get

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

and after tidying this up, we get the actual quadratic formula which is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Example**

Solve, using the quadratic formula

$$3x^2 + 11x + 8 = 0$$

Page 21 Ex 2E

***Historical background to the Quadratic Formula***

Quadratic equations were known and dealt with by Babylonians, as early as 1800BC; in the Sulba Sutras (sutra text relating to altar construction in Hinduism and the main source of our understanding of Indian mathematics from the Vedic period – 1500-500BC) the quadratics of the form

$ax^2 = c$                       and                       $ax^2 + bx = c$  were explored using geometric methods.

Al-Khwarizmi also, as previously mentioned, (in relation to the technique of ‘completing the square’) worked with quadratic equations.

As you have seen above, the quadratic formula for solving equations can be easily derived from the completed square form of a quadratic.

## Quadratics – Curve Sketching

Quadratics take the form  $y = ax^2 + bx + c$

The graph of this equation is called *parabola*.

As previously mentioned, the parabola will be  $\cup$  shaped if  $a > 0$ , and  $\cap$  shaped if  $a < 0$ .

You now need to find out if the curve crosses or touches the  $x$  axis or does neither. The easiest way to see this is to look at the quadratic formula, and especially part under the square root

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

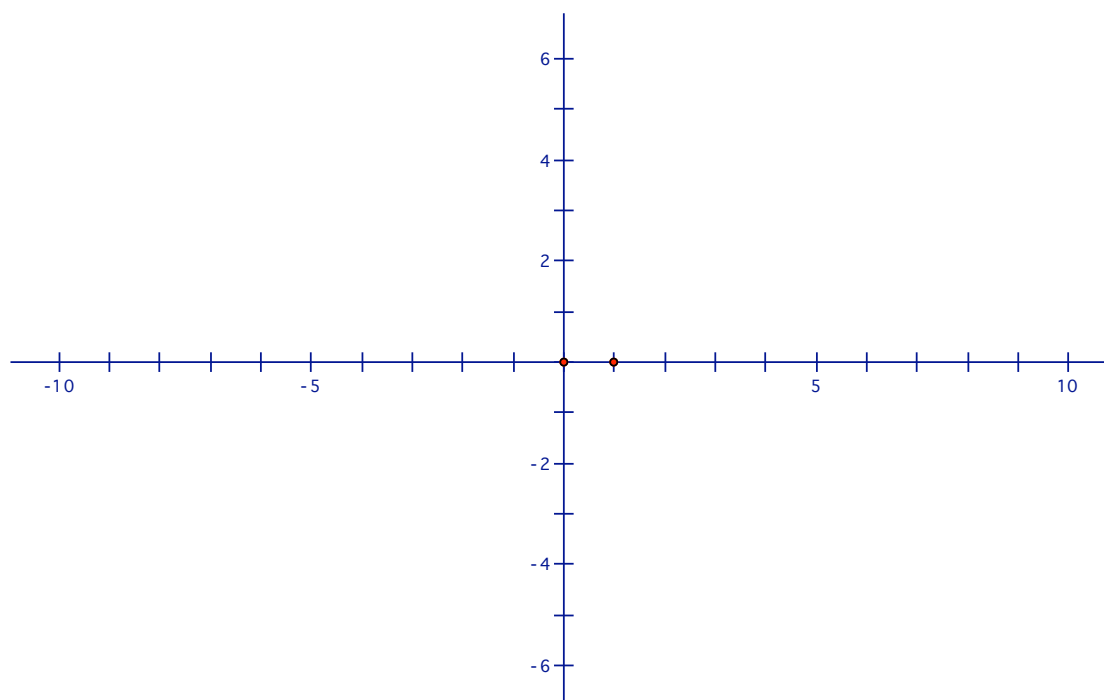
This part,  $b^2 - 4ac$ , is called the *discriminant*.

If...	Then...	How many roots...
$b^2 - 4ac > 0$	Curve intersects x axis	2 roots
$b^2 - 4ac = 0$	Curve touches x axis	1 repeated root
$b^2 - 4ac < 0$	Curve misses x axis	No real roots

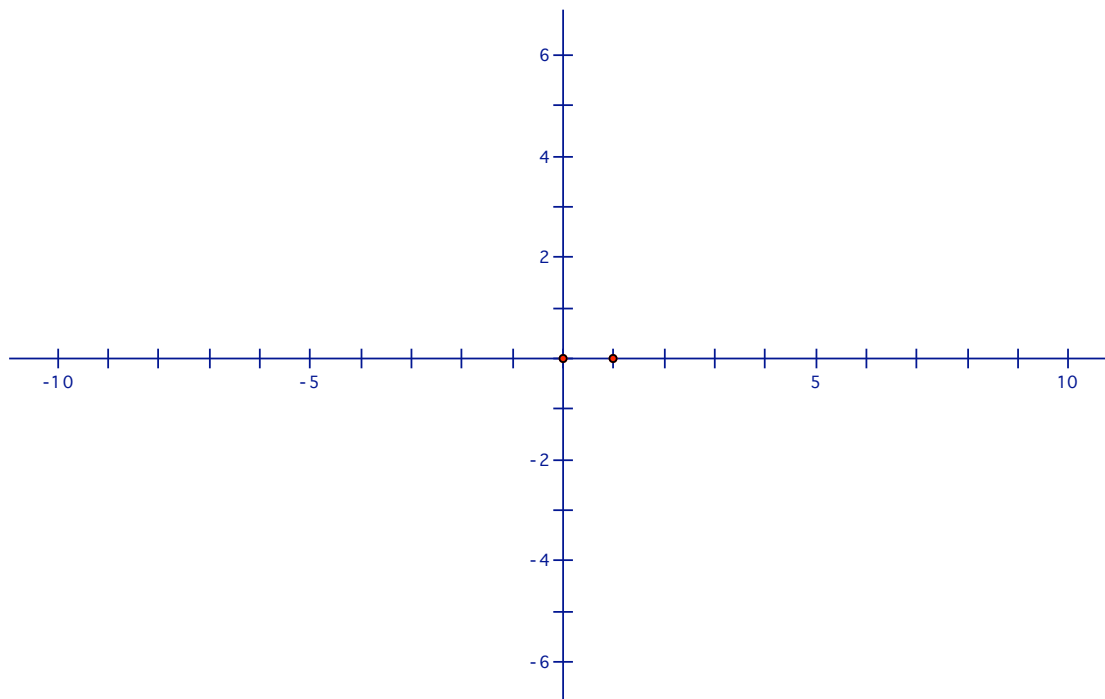
### Examples

Sketch the following graphs, indicating the coordinates of any points where the curve meets the axis.

1.  $y = x^2 - 4x + 3$



2.  $y = -x^2 + 4x - 5$



3. Find the values of  $k$  for which  $x^2 + kx + 9 = 0$  has equal roots.

Note: When you are solving quadratics you should always try and factorise first.  
How will you know whether an expression can be factorised?

Well you know that if discriminant is positive, i.e.  $b^2 - 4ac > 0$ , it has an real roots,  
so that would mean that it can also be factorised.

## *A short history of parabola*

Parabola gets the name from Greek *παράβολή*. It was first studied by the Greek mathematicians Menaechmus (380-320BC), Apollonius (262-190BC) and Pappus (290-350). Menaechmus found the parabola as a side-product to one of the problems he was interested in.

One of the most famous problems in mathematics is trying to duplicate the cube. Perhaps its origin was mystical – it appears that Athenians, who were suffering the great outbreak of plague went to an Oracle at Delos to ask for the cure. The Oracle advised them that altar to the god Apollo should be doubled – having a cube for the altar this meant that the new altar should be double the size of the first. Although there was (and is) some confusion as to whether the oracle meant ‘double’ to mean linear dimension (scale) or size (volume) or simply to put one cube upon another (totally non-scientific interpretation), the problem of constructing a double of a cube with a given side remains geometrically unsolved.

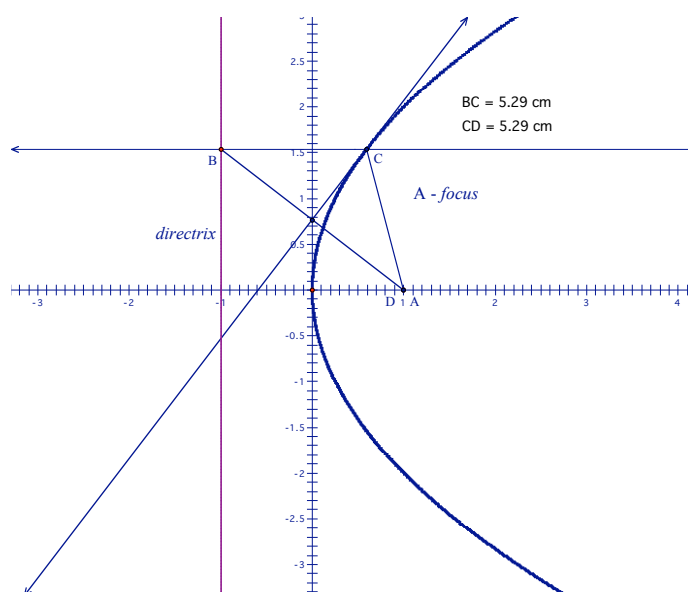
Although the problem may sound simple, it is in fact impossible to solve by a Euclidean construction (a Euclidean construction is one in which one can use only an unmarked straight edge and a collapsible compass, as was proved by Wantzel 1814-1848, French mathematician and engineer in 1837). But by trying to find the solution to this problem over the centuries, many interesting things came up – and the case in point is Menaechmus’ discovery of parabola.

He attempted to solve the problem of  $x^3 = 2$  by geometrical methods, and considered two parabolas,  $x^2 = y$  and  $y^2 = 2x$ . Euclid (~325BC – ~265BC) wrote about the curve, but as yet it did not have the name. Then Apollonius (~262BC - ~190BC) named it in his famous book on *Conics*.

Parabola is a curve which is defined as a set of all points in a plane equidistant from a given line and a given point. The given line is called directrix (giving direction) and a given point is called focus of the parabola.

You can generate a parabola by stretching a piece of an rope, one of its ends moving across the directrix, and having the other as a stationary point at the focus, and always making sure that the distance between the focus and the directrix remains the same whilst changing the total length of the rope.

See how it works at [www.mathsisgoodforyou.com/applets/parabola](http://www.mathsisgoodforyou.com/applets/parabola).



## Simultaneous Equations

1. Solve, using the method of elimination

$$8p + 3q = 11$$

$$6p - 2q = 21$$

2. Solve, using the method of substitution

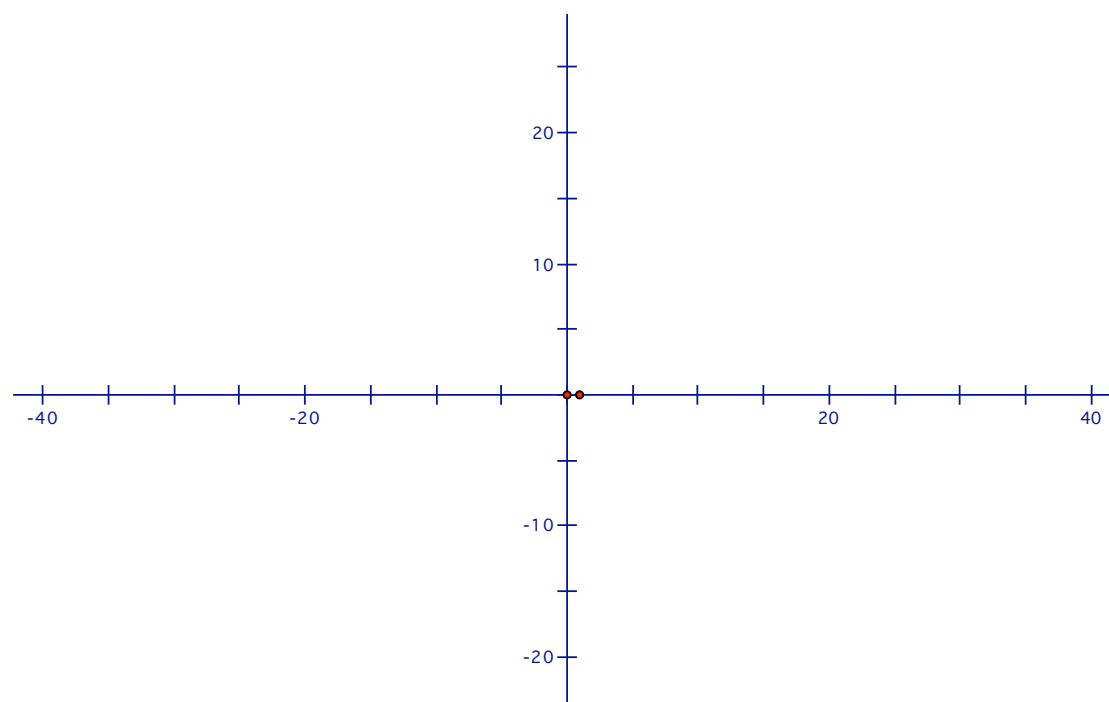
$$5a + 9b = 1$$

$$a - 15b = 10$$

Page 26 Ex 3A and 3B

### Solving one linear and one quadratic equation simultaneously

Now, sketch the graph of  $y = 3x^2 - 5$  and  $y = 2 + 4x$



How many points of intersection are there?

Finding the points of intersection means solving the two equations simultaneously.

**Example**

Solve the simultaneous equations

$$4x - y = 4$$

$$x^2 + 5y = 1$$

# Inequalities

## *Historical Introduction*

Signs for inequalities developed only in the 16<sup>th</sup> and 17<sup>th</sup> century. The signs for ***less than*** ( $<$ ), and ***greater than*** ( $>$ ) appeared first in a Thomas Hariot '*Praxis*' in 1631. Hariot was an English mathematician, author and astronomer, most famous for his development of algebra in England and the English speaking world.

The signs ***less or equal than*** ( $\leq$ ) and ***greater or equal than*** ( $\geq$ ) appeared almost a century later, in 1734, in a book by Pierre Bouguer, a French mathematician, astronomer and naval architect.

Remember to treat the inequality signs very gently. They are, as you see, reasonably old and fragile... When dealing with an inequality, treat it like an equation as long as you don't multiply it (in any conceivable way) with -1. In that case, turn the inequality sign the other way around.

## Examples

When solving 2 inequalities it is useful to use a number-line.

1. Find the set of values for which  $3x - 5 < x + 8$  and  $5x > x - 8$



2. Find the set of values for which  $x - 5 > 1 - x$  and  $15 - 3x > 5 + 2x$



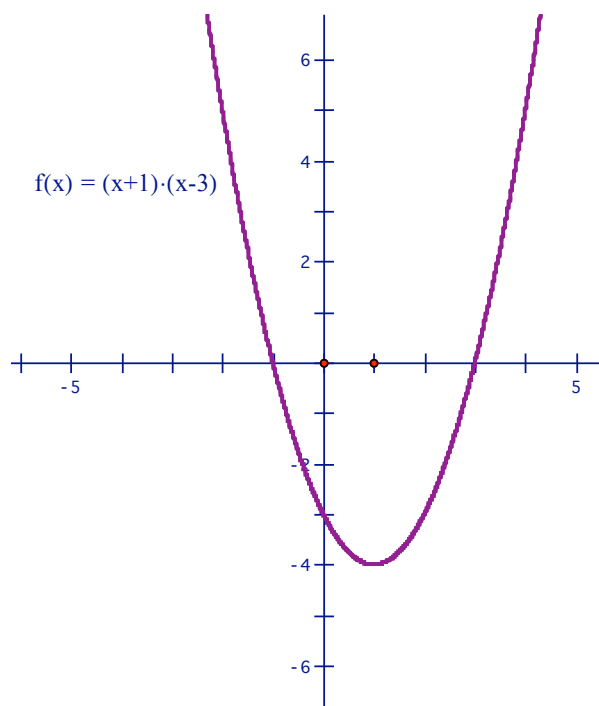
3. Find the set of values for which  $4x + 7 > 3$  and  $17 < 11 + 2x$



## Quadratic Inequalities

The easiest way to deal with inequalities is to treat them as equations. As long as you don't multiply by  $(-1)$  nothing should change; once you do this however, you need to swap the sign with its opposite ( $<$  with  $>$  and vice-versa).

### Examples



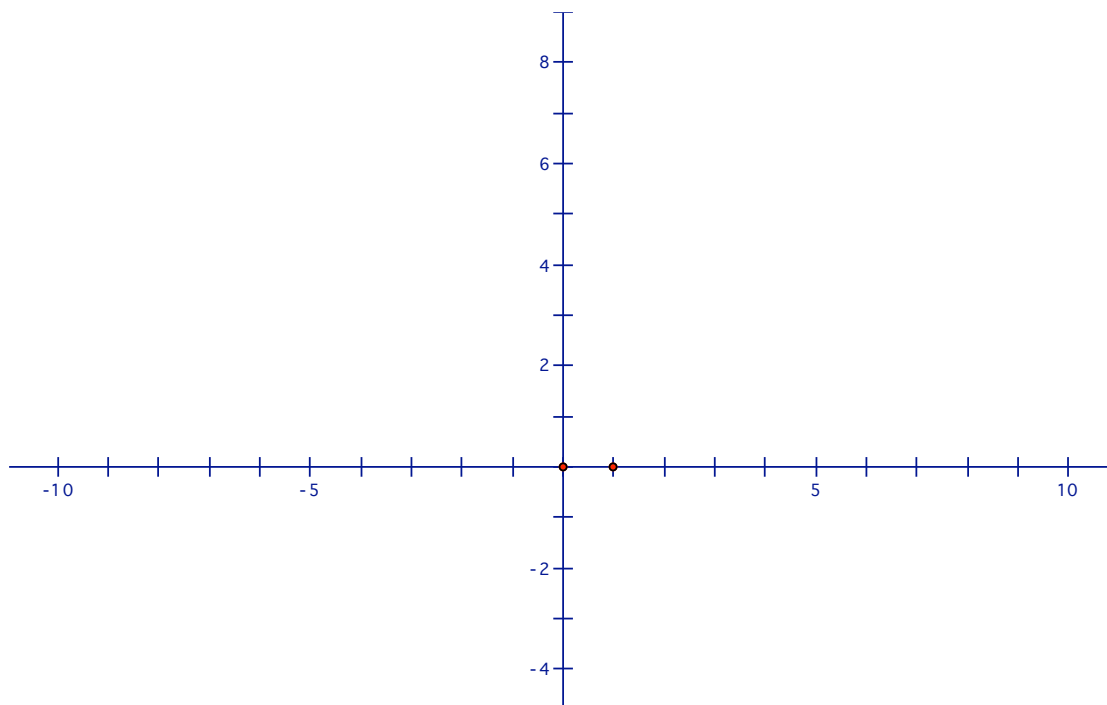
1. Find the set of values for which  $x^2 - 2x - 3 > 0$ .

First factorise the equation and then sketch the graph.  $(x+1)(x-3) > 0$

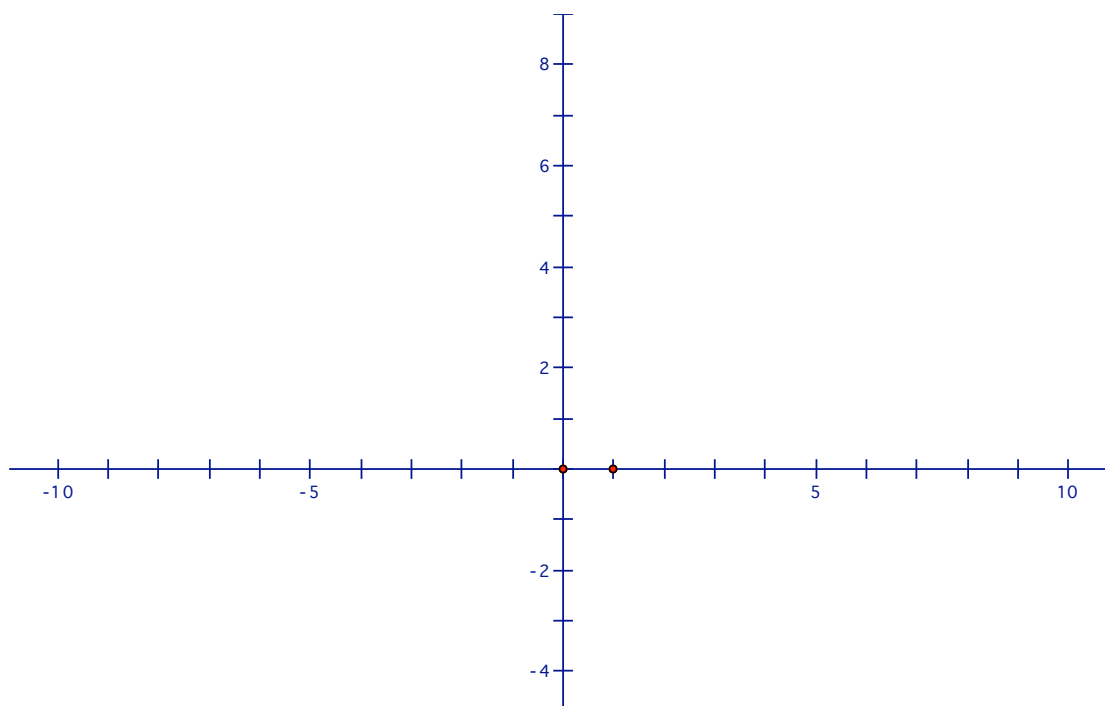
When you have the sketch, you can see that the values for which this values of  $y$  are positive – above the  $x$  axis are when

$$x < -1, x > 3$$

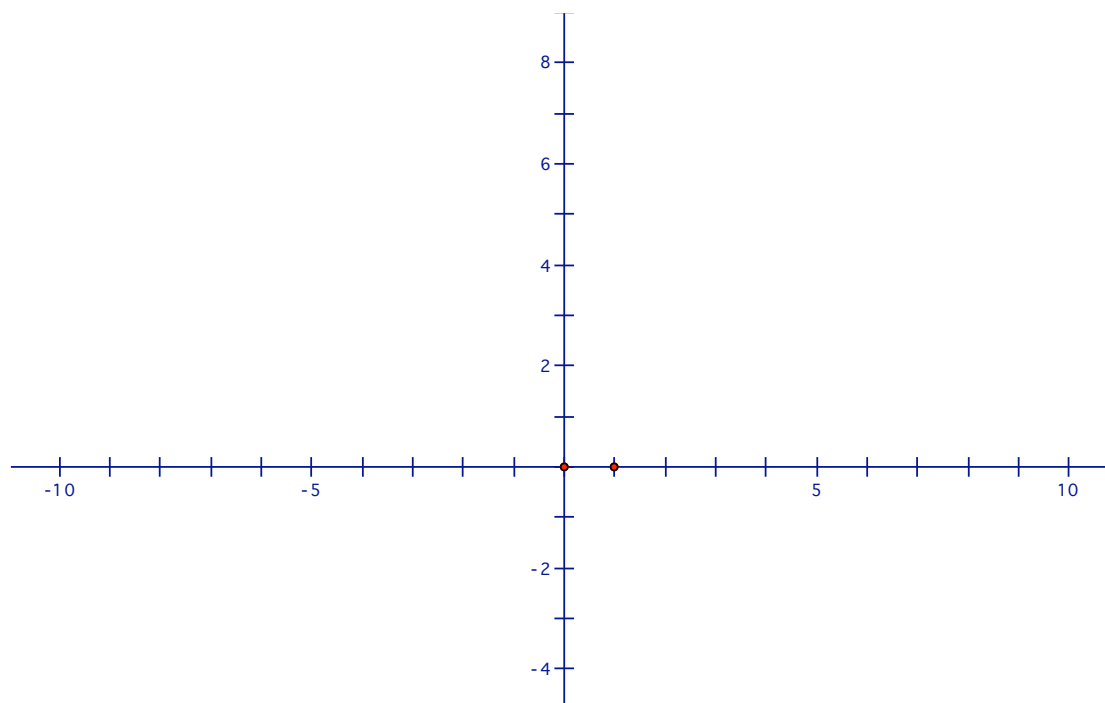
2. Find the set of values for which  $x^2 - 5x + 6 \leq 0$



3. Find the set of values of  $x$  for which  $3x + 4 > x^2$



4. Find the set of values for which  $3x + 4 > x^2$  and  $3x - 3 > 2x + 3$



## Answers

### Indices page 3

1. 5, 2. -2, 3.  $1/8$ , 4. -1  
5. 1, 6.  $5/9$ , 7.  $1/2$  8.  $3/2$

### Surds page 5

1.  $\sqrt{10}$ , 2.  $\sqrt{7}$ , 3.  $6\sqrt{2}$ , 4.  $7\sqrt{2}$

### Surds page 6

1.  $\frac{\sqrt{5}}{10}$ , 2.  $\frac{\sqrt{5}-3}{2}$ , 3.  
 $\frac{3+\sqrt{5}+3\sqrt{2}+\sqrt{10}}{4}$

### Factorising quadratics – top of page 8

$$3x^3 + 3x^2 - x - 4$$

### Factorisation page 9

1.  $x(8x+5)$   
2.  $5x(5x-3)(x+1)$   
3.  $(3x-2)(3x+2)$

### Solving quadratics page 10

1.  $x = \pm 2$       2.  $x_1 = -3$   
 $x_2 = 12$

3.  $x_1 = -2$   
 $x_2 = \frac{9}{2}$       4.  $x_1 = -2$   
 $x_2 = 3$

### Examples on completing the square page 12

1.  $(x+4)^2 - 16 = 0$   
2.  $(x+6)^2 - 36 = 0$   
3.  $2(x-\frac{5}{2})^2 - \frac{25}{2} = 0$

$$4. (x+\frac{3}{2})^2 - \frac{9}{4} = 0$$

### Solving quadratics using quadratic formula page 15

$$x_1 = 1$$

$$x_2 = \frac{8}{3}$$

### Quadratics – curve sketching pages 17 & 18

1.  $x_1 = 3$   
 $x_2 = 1$

2. discriminant is  $< 0$  and so there are no real solutions

3.  $k = \pm 6$

### Simultaneous equations page 20

1.  $x = \frac{5}{2}, y = -3$

2.  $x = \frac{5}{4}, y = -\frac{7}{12}$

$x = -1, y = -2$   
solving simultaneously  $x = \frac{7}{3}, y = \frac{34}{3}$

$x = 1, y = 0$   
example page 21  $x = -21, y = -88$

### Inequalities – pages 22 & 23

1.  $-\frac{1}{2} < x < \frac{13}{2}$ , 2. no common

solution, 3.  $x > 3$

### Quadratic inequalities pages 24 & 25

2.  $2 \leq x \leq 3$

3.  $-1 < x < 4$

4. no common solution

## ***Algebra test 1***

The first test covers first three chapters from the C1 book as well as all the examples done in the lessons and given here. The majority of questions will have algebraic manipulation so please see me during my free periods if you need some extra help or are unsure about certain topics.

Good way of revising is to complete all the mixed exercises which can be found at the end of each chapter.

You can download this from <http://www.mathsisgoodforyou.com/ALevelMaths.htm>  
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